technical monograph 30

Fundamentals of Valve Sizing for Liquids

Marc Riveland

Senior Research Engineer

FISHER®

FISHER-ROSEMOUNT™

Fundamentals of Valve Sizing for Liquids

Notations

A = cross sectional flow area
C_v = flow coefficient
d = diameter of valve inlet
D = diameter of adjacent piping
F_d = valve style modifier
F_F = critical pressure ratio
F_L = pressure recovery coefficient
F_p = piping correction factor
F_R = Reynolds number factor
g = gravitational acceleration
g_c = gravitational constant
G = liquid specific gravity
H_l = available head loss
K_B = Bernoulli coefficient
K_l = available head loss coefficient
K_m = pressure recovery coefficient
N_1 = units factor
N_2 = units factor
N_4 = units factor
P = pressure
P_c = absolute thermodynamic critical pressure
q = heat transferred out of fluid
Re_v = Reynolds number
r_c = critical pressure ratio
Q = flow rate
U = internal energy of fluid
V = velocity
w = shaft work done by (or on) fluid
Z = elevation
p = fluid density
ν = kinematic viscosity

General Subscripts:

1 = upstream
2 = downstream
v = vapor
vc = vena contracta

Introduction

Valves are selected and sized to perform a specific function within a process system. Failure to perform that given function, whether it is controlling a process variable or simple on/off service, results in higher process costs. The sizing function thus becomes a critical step to successful process operation.

This paper focuses on correctly sizing valves for liquid service.

Liquid Sizing Equation Background

This section presents the technical substance of the liquid sizing equations. The value of this lies in not only a better understanding of the sizing equations, but also in knowledge of their intrinsic limitations and relationship to other flow equations and conditions.

The flow equations used for sizing have their roots in the fundamental equations which describe the behavior of fluid motion. The two principle equations are the energy equation and the continuity equation.

The energy equation is equivalent to a mathematical statement of the first law of thermodynamics. It accounts for the energy transfer and content of the fluid. For an incompressible fluid (e.g. a liquid) in steady flow, this equation can be written as:

$$\left(\frac{V^2}{2g_c} + \frac{P}{\rho} + gZ\right) - w + q + U = constant \tag{1}$$

where all terms are defined in the nomenclature section. The three terms in parenthesis are all mechanical (or available) energy terms and carry a special significance. These quantities are all capable of directly doing work. Under certain conditions more thoroughly described later, this quantity may also remain constant:

$$\frac{V^2}{2g_c} + \frac{P}{\rho} + gZ = constant \tag{2}$$

This equation can be derived from purely kinematic methods (as opposed to thermodynamic methods) and is known as Bernoulli's equation.

The other fundamental equation which plays a vital role in the sizing equation is the continuity equation. This is the mathematical statement of conservation of the fluid mass. For steady flow conditions (one–dimensional) this equation is written as:

$$\rho VA = constant \tag{3}$$

Using these fundamental equations, we can examine the flow through a simple fixed restriction such as that shown in figure 1. We will assume the following for the present:

1. The fluid is incompressible (a liquid)
2. The flow is steady

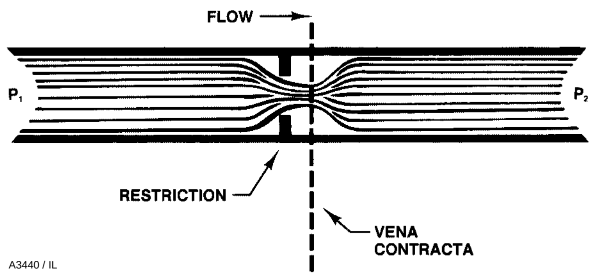


Figure 1. Flow through a Simple Fixed Restriction

3. The flow is one–dimensional
4. The flow can be treated as inviscid (explained later)
5. No change of fluid phase occurs

As seen in figure 1, the flow stream must contract to pass through the reduced flow area. The point along the flow stream of minimum cross sectional flow area is the vena contracta. The flow processes upstream of this point and downstream of this point differ substantially, so it is convenient to consider them separately.

The process from a point several pipe diameters upstream of the restriction to the vena contracta is very nearly ideal for practical intents and purposes (thermodynamically isentropic). Under this constraint, Bernoulli's equation applies and we see that no mechanical energy is lost —it merely changes from one form to the other. Furthermore, changes in elevation are negligible since the flow stream centerline changes very little, if at all. Thus, energy contained in the fluid simply changes from pressure to kinetic. This is quantified when considering the continuity equation. As the flowstream passes through the restriction, the velocity must increase inversely proportional to the change in area. For example, from equation 3:

$$V_{vc} = \frac{(constant)}{A_{vc}} \tag{4}$$

Using upstream conditions as a reference, this becomes:

$$V_{vc} = V_1\left(\frac{A_1}{A_{vc}}\right) \tag{5}$$

Thus, as the fluid passes through the restriction, the velocity increases. Applying equation 2 and neglecting elevation changes (again using upstream conditions as a reference):

$$\frac{\rho V_1^{\,2}}{2g_c} + P_1 = \frac{\rho V_{vc}^{\,2}}{2g_c} + P_{vc} \tag{6}$$

Inserting equation 5 and rearranging, results in:

$$P_{vc} = P_1 - \frac{\rho V_1^{\,2}}{2g_c}\left[\left(\frac{A_1}{A_{vc}}\right)^2 - 1\right] \tag{6a}$$

Thus, at the point of minimum cross sectional area, we see that fluid velocity is at a maximum (from equation 5) and fluid pressure is at a minimum (from equation 6).

The process from the vena contracta point to a point several diameters downstream is not ideal and equation 2 no longer applies. By arguments similar to above, it can be reasoned (from the continuity equation) that as the original cross sectional area is restored, the original velocity is also restored. Because of the non–idealities of this process, however, the total mechanical energy is not restored. A portion of it is converted into heat which is either absorbed by the fluid itself or dissipated to the environment. Let us consider equation 1 applied from several diameters upstream of the restriction to several diameters downstream of the restriction:

$$U_1 + \frac{V_1^{\,2}}{2g_c} + \frac{P_1}{\rho} + \frac{gZ_1}{g_c} = q = U_2 + \frac{V_2^{\,2}}{2g_c} + \frac{P_2}{\rho} + \frac{gZ_2}{g_c} = w \tag{7}$$

No work is done across the restriction, so the work term drops out. The elevation changes are negligible, so the respective terms cancel each other. We can combine the thermal terms into a single term, H_l:

$$\frac{\rho V_1^{\,2}}{2g_c} + P_1 = \frac{\rho V_2^{\,2}}{2g_c} + P_2 + H_I \tag{8}$$

The velocity was restored to its original value so that equation 8 reduces to:

$$P_1 = P_2 + H_I \tag{9}$$

Thus, the pressure decreases across the restriction and the thermal terms (internal energy and heat lost to the surroundings) increase.

Losses of this type are generally proportional to the square of the velocity (references 1 and 2), so it is convenient to represent them by the following equation:

$$H_I = K_I\,\frac{\rho V^2}{2} \tag{10}$$

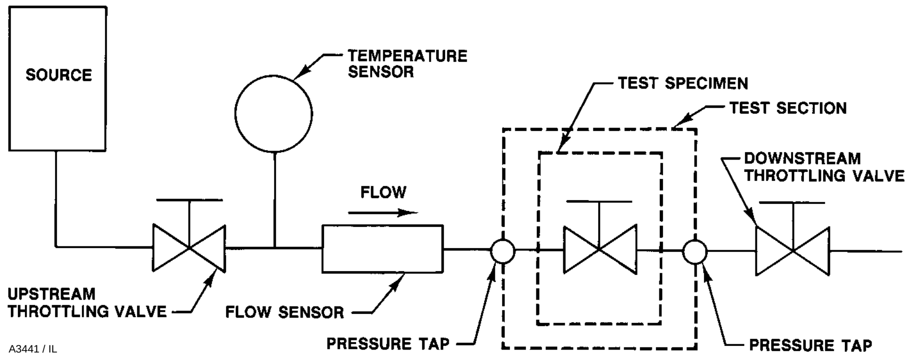


Figure 2. ISA Flow Test Piping Configuration

In this equation the constant of proportionality, K_I, is called the available head loss coefficient, and is determined by experiment.

From equations 9 and 10 it can be seen that the velocity (at location 2) is proportional to the square root of the pressure drop. Volume flow rate can be determined knowing the velocity and corresponding area at any given point so that:

$$Q = V_2 A_2 = \sqrt{\frac{2(P_1 - P_2)}{\rho K_I}}\, A_2 \qquad (11)$$

Now, letting:

$$\rho = G \rho_w$$

and, defining:

$$C_v = A_2 \sqrt{\frac{2}{\rho_w K_I}} \qquad (12)$$

where G is the liquid specific gravity, equation 11 may be rewritten as:

$$Q = C_v \sqrt{\frac{P_1 - P_2}{G}} \qquad (13)$$

Equation 13 constitutes the basic sizing equation used by the control valve industry, and provides a measure of flow in gallons per minute (GPM) when pressure in pounds per square inch is used. Sometimes it may be desirable to work with other units of flow or independent flow variables (pressure, density, etc.). The equation fundamentals are the same for such cases and only constants or form are different. Reference 3 provides an excellent summary of the variant forms of the liquid flow equation.

Determination of Flow Coefficients

Rather than experimentally measure K_I and calculate C_v, it is more straight–forward to measure C_v directly.

In order to assure uniformity and accuracy, the procedures for both measuring flow parameters and use in sizing are addressed by industrial standards. The currently accepted standards are sponsored by the Instrument Society of America (ISA) as given in reference 4.

Measurement of C_v and related flow parameters is covered extensively in reference 4 and is reviewed only briefly here.

The basic test system configuration is shown in figure 2. Specifications, accuracies and tolerances are given for all hardware installation and data measurements such that coefficients can be calculated to an accuracy of approximately ±5%. Fresh water at approximately 68^{o}F is circulated through the test valve at specified pressure differentials and inlet pressures. Flow rate, fluid temperature, inlet and differential pressure, valve travel and barometric pressure are all measured and recorded. This yields sufficient information to calculate the following sizing parameters (the next section explains the meaning and use of these factors):

Flow coefficient (C_v)
Pressure recovery coefficient (F_L)
Piping correction factor (F_p)
Reynolds number factor (F_R)

In general, each of these parameters depends on the valve style and size, so multiple tests must be performed accordingly. These values are then published by the valve manufacturer for use in sizing.

Basic Sizing Procedure

The procedure by which valves are sized for normal, incompressible flow is straightforward. Again, to insure uniformity and consistency, a standard exists which delineates the equations and correction factors to be employed for a given application (reference 5).

The simplest case of liquid flow application involves the basic equation developed earlier. Rearranging equation 13 so that all of the fluid and process related variables are on the right side of the equation, we arrive at an expression for the valve Cv required for the particular application:

$$C_v = \frac{Q}{\sqrt{\frac{P_1 - P_2}{G}}} \tag{14}$$

It is important to realize that valve size is only one aspect of selecting a valve for a given application. Other considerations include valve style and trim characteristic. Discussion of these features falls outside the scope of this monograph. Other sources, such as references 6 and 7 make a thorough presentation.

Once a valve has been selected and C_V is known, the flow rate for a given pressure drop, or the pressure drop for a given flow rate, can be predicted by substituting the appropriate quantities into equation 13.

Many applications fall outside the bounds of the basic liquid flow applications just considered. Rather than develop special flow equations for all of the possible deviations, it is possible (and preferred) to account for different behavior with the use of simple correction factors. These factors, when incorporated, change the form of equation 13 to the following (reference 5):

$$Q = (N_1 F_p F_R)\, C_v \sqrt{\frac{P_1 - P_2}{G}} \tag{15}$$

All of the additional factors in this equation are explained in the following sections.

Choked Flow

Equation 13 would imply that, for a given valve, flow could be continually increased to infinity by simply increasing the pressure differential across the valve. In reality, the relationship given by this equation holds for only a limited range. As the pressure differential is increased, a point is reached where the realized mass flow increase is less than expected. This phenomenon continues until no additional mass flow increase occurs in spite of increasing the pressure differential (figure 3). This condition of limited maximum mass flow is known as choked flow. To understand more about what is occurring and how to correct for it when sizing valves, it is necessary to return to some of the fluid flow basics discussed earlier.

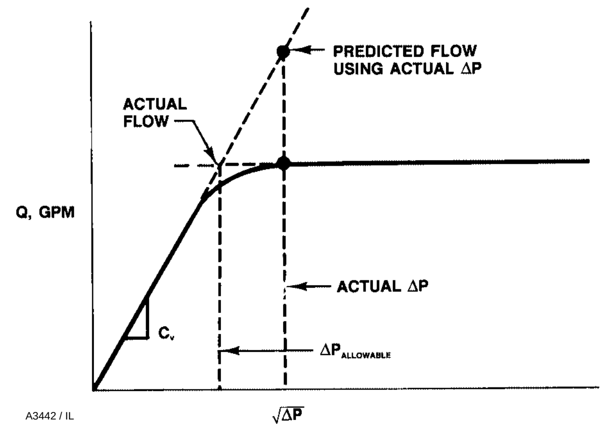


Figure 3. Typical Flow Curve Showing Relationship Between Flow Rate Q and Imposed Pressure Differential ΔP

Recall that as a liquid passes through a reduced cross–sectional area, velocity increases to a maximum and pressure decreases to a minimum. As the flow exits, velocity is restored to its original value while the pressure is only partially restored, thus creating a pressure differential across the device. As this pressure differential is increased, the velocity through the restriction increases (thus increasing flow) and the vena contracta pressure decreases. If a sufficiently large pressure differential is imposed on the device, the minimum pressure may decrease to or below the vapor pressure of the liquid under these conditions. When this occurs, the liquid becomes thermodynamically unstable and partially vaporizes. The fluid now consists of a mixture of liquid and vapor which is no longer incompressible.

While the exact mechanisms of liquid choking are not fully confirmed, there are parallels between this and critical flow in gas applications. In gas flows the flow becomes critical (choked) when the fluid velocity is equal to the acoustic wave speed at that point in the fluid. Pure incompressible fluids have very high wave speeds so, practically speaking, they do not choke. Liquid/gas or liquid/vapor mixtures, however, typically have very low acoustic wave speeds (actually lower than that for a pure gas or vapor) so that it is possible for the mixture velocity to equal the sonic velocity and choke the flow.

Another way of viewing this phenomenon is to consider the density of the mixture at the vena contracta. As the pressure decreases, the density of the vapor phase, and hence the mixture, decreases. Eventually this decrease in density of the fluid offsets any increase in the velocity of the mixture, to the point where no additional mass flow is realized.

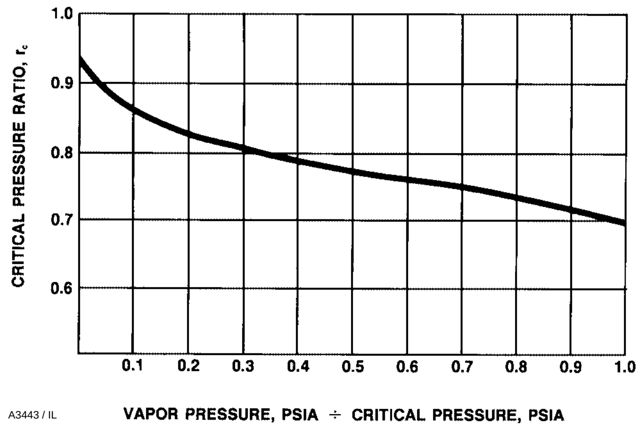


Figure 4. Generalized r_c Curve

It is necessary to account for the occurrence of choked flow during the sizing process to insure against undersizing a valve. In other words, we need to know the maximum flow rate a valve can handle under a given set of conditions. To this end, a procedure was developed (reference 8) which combines the control valve pressure recovery characteristics with the thermodynamic properties of the fluid to predict the maximum usable pressure differential, that is, the pressure differential at which the flow just chokes.

Figure 4. Generalized r_c Curve

A pressure recovery coefficient can be defined as:

$$K_m = \frac{P_1 - P_2}{P_1 - P_{vc}} \tag{16}$$

Under choked flow conditions it is established in reference 8 that:

$$P_{vc} = r_c P_v \tag{17}$$

The vapor pressure, P_v, is determined at inlet temperature since the temperature of the liquid does not change appreciably between the inlet and the vena contracta. The term r_c is known as the critical pressure ratio and is another thermodynamic property of the fluid. While it is actually a function of each fluid and the prevailing conditions, it has been established that data for a variety of fluids can be generalized according to figure 4 (references 5 and 8) or the following equation (reference 6) without significantly compromising overall accuracy:

$$r_c = F_F = 0.96 - 0.28\sqrt{\frac{P_{vc}}{P_c}} \tag{19}$$

The value of K_m is determined individually by test for each valve style and accounts for the pressure recovery characteristics of the valve.

By rearranging equation 16, the pressure differential at which the flow chokes can be determined is called the allowable pressure differential:

$$(P_1 - P_2)_{allowable} = K_m(P_1 - r_c\ P_v) \tag{20}$$

When this allowable pressure differential is used in equation 13, the choked flow rate for the given valve will result. If this flow rate is less than the required service flow rate, the valve is undersized. It is then necessary to select a larger valve and repeat the calculations using the new values for C_v and K_m.

The equations supplied in the sizing standard (reference 5) are in essence the same as those presented in this paper, except the nomenclature has been changed. In this case:

$$Q_{max} = N_1 F_L C_v \sqrt{\frac{P_1 - F_F P_v}{G}} \tag{21}$$

where:

F_L = $\sqrt{K_m}$

F_F = rc

N_1 = units factor

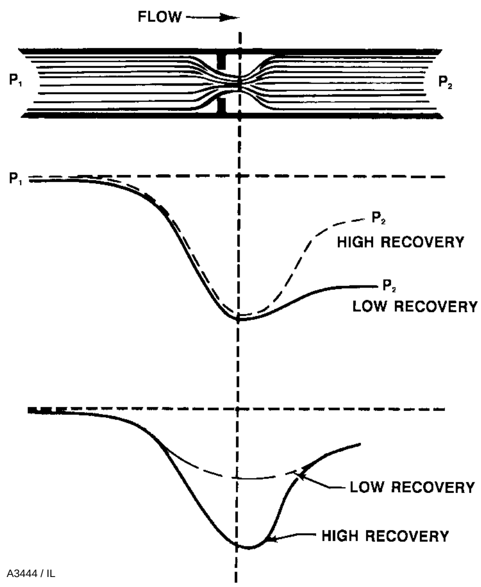
Cavitation

Closely associated with the phenomenon of choked flow is the occurrence of cavitation. Simply stated, cavitation is the formation and collapse of cavities in the flowing liquid. It is of special concern in sizing control valves because left unchecked it can produce unwanted noise, vibration, and material damage.

As discussed earlier, vapor can form in the vicinity of the vena contracta when the local pressure drops below the vapor pressure of the liquid. If the outlet pressure seen by the mixture as it exits the control valve is greater than the vapor pressure, the vapor phase will be thermodynamically unstable and will revert to a liquid. The entire liquid–vapor–liquid phase change process is known as cavitation, although it is the vapor–to–liquid phase change which is the primary source of the damage. During this phase change, a mechanical attack occurs on the material surface in the form of high velocity micro–jets and shock waves. Given sufficient intensity, proximity, and time, this attack can remove material to the point where the valve no longer retains its functional or structural integrity. Figure 5 shows an example of such damage.


W1350 / IL

Figure 5. Typical Cavitation Damage

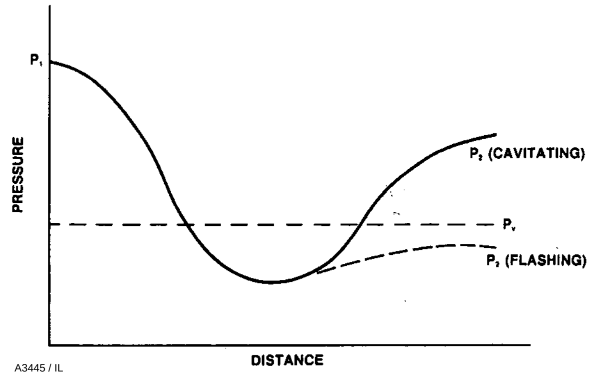


A3444 / IL

Figure 6. Comparison of High and Low Recovery Valves

Because cavitation and the damage due to it are complex processes, accurate prediction of key events such as damage, noise, and vibration level is difficult. Thus sizing valves for cavitation conditions requires special considerations.

The concept of pressure recovery plays a key role in characterizing a valve's suitability for cavitation service. A valve which recovers a significant percentage of the pressure differential from inlet to the vena contracta is appropriately termed a high recovery valve. Conversely, if only a small percent is recovered it's classified as a low recovery valve. These two are contrasted in figure 6. If identical pressure differentials are imposed on a high recovery valve and a low recovery valve, all other things being equal, the high recovery valve will have a relatively low vena contracta pressure. Thus, under the same conditions, the high recovery valve will more likely cavitate. On the other hand, if flow through each is such that the inlet and



A3445 / IL

Figure 7. Pressure Profiles for Flashing and Cavitating Flows

vena contracta pressures are equal the low recovery valve will have the lower collapse potential (P_2–P_{VC}), so that cavitation intensity will generally be less.

Thus, it is apparent that the lower pressure recovery devices are more suited for cavitation service.

The possibility of cavitation occurring in any liquid flow application should be investigated by checking for the following two conditions:

1. The service pressure differential is approximately equal to the allowable pressure differential, and

2. The outlet pressure is greater than the vapor pressure of the fluid.

If both of these conditions are met the possibility exists that cavitation will occur. Because of the potentially damaging nature of cavitation, sizing a valve in this region is not recommended. Special purpose trims and products to control cavitation should be considered. Because of the great diversity in the design of this equipment it is not possible to offer general guidelines for sizing them. Refer to specific product literature for more information.

Flashing

Flashing shares some common features with choked flow and cavitation in that the process begins with vaporization of the liquid in the vicinity of the vena contracta. However, in flashing applications, the pressure downstream of this point never recovers to a value which exceeds the vapor pressure of the fluid so that the fluid remains in the vapor phase. Schematic pressure profiles for flashing and cavitating flow are contrasted in figure 7.

Flashing is of concern not only because of its ability to limit flow through the valve, but also because of the highly erosive nature of the liquid–vapor mixture. Typical flashing damage is smooth and polished in

Figure 8. Typical Flashing Damage

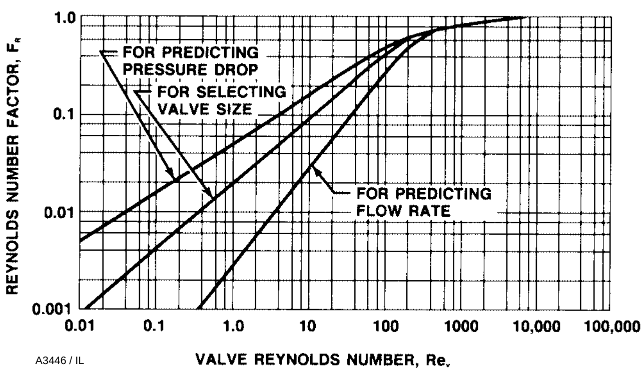


Figure 9. Viscous Flow Correction Factors

appearance (figure 8) in stark contrast to the rough, cinderlike appearance of cavitation (figure 5).

If $P_2 < P_v$, or there are other service conditions to indicate flashing, the standard sizing procedure should be augmented with a check for choked flow. Furthermore, suitability of the particular valve style for flashing service should be established with the valve manufacturer.

Viscous Flow

One of the assumptions implicit in the sizing procedures presented to this point is that of fully developed, turbulent flow. Turbulent flow and laminar flow are flow regimes which characterize the behavior of flow. In laminar flow all fluid particles move parallel to one another in an orderly fashion and with no mixing of the fluid. Conversely, turbulent flow is highly random in terms of local velocity direction and magnitude. While there is certainly net flow in a particular direction, instantaneous velocity components in all directions are superimposed on this net flow. Significant fluid mixing occurs in turbulent flow. As is true of many physical phenomena, there is no distinct line of demarcation between these two regimes, so a third regime of transition flow is sometimes recognized.

The physical quantities which govern this flow regime are the viscous and inertial forces, the ratio of which is known as the Reynolds number. When the viscous forces dominate (a Reynolds numbers below 2000) the flow is laminar, or viscous. If the inertial forces dominate (a Reynolds number above 3000) the flow is turbulent, or inviscid.

Consideration of these flow regimes is important because the macroscopic behavior of the flow changes when the flow regime changes. The primary behavior characteristic of concern in sizing is the nature of the available energy losses. In earlier discussion it was asserted that, under the assumption of inviscid flow, the available energy losses were proportional to the square of the velocity.

In the laminar flow regime, these same losses are linearly proportional to the velocity; in the transitional regime, these losses tend to vary. Thus, for equivalent flow rates, the pressure differential through a conduit or across a restriction will be different for each flow regime.

To compensate for this effect (the change in resistance to flow) in sizing valves a correction factor was developed (reference 9). The required C_v can be determined from the following equation:

$$C_{v_{req'd}} = F_R \ C_{v_{rated}} \qquad (22)$$

The factor F_R is a function of the Reynolds number and can be determined from a simple nomograph procedure (reference 10), or by calculating the Reynolds number for a control valve from the following equation and determining F_R from figure 9 (reference 9).

$$\mathrm{Re}_v = \frac{N_4 F_d Q}{\nu F_L 1/2 \ C_v 1/2}\left[\frac{1}{N_2}(F_L)^2\left(\frac{C_v}{d^2}\right)^2 + 1\right]^{1/4} \qquad (23)$$

To predict flow rate or resulting pressure differential, the required flow coefficient is used in place of the rated flow coefficient in the appropriate equation.

When a valve is installed in a field piping configuration which is different than the specified test section, it is necessary to account for the effect of the altered piping on flow through the valve. (Recall that the standard test section consists of a prescribed length of straight pipe up and downstream of the valve.) Field installation may require elbows, reducers, and tees, which will induce additional losses immediately adjacent to the valve. To correct for this situation, two factors are introduced: F_p and F_{lp}. The former is used to correct the flow equation when used in the incompressible range, while the latter is used in the choked flow range. The expressions for these factors are:

$$F_p = \left[\frac{\Sigma K}{N_2} \left(\frac{C_v}{d^2} \right)^2 + 1 \right]^{-1/2} \tag{24}$$

$$F_{Ip} = F_L \left[\frac{F_L{}^2 K_I}{N_2} \left(\frac{C_v}{d^2} \right)^2 + 1 \right]^{-1/2} \tag{25}$$

The term ∑K in equation 24 is the sum of all loss coefficients of all devices attached to the valve and the inlet and outlet Bernoulli coefficients. Bernoulli coefficients are coefficients to the velocity head term in the energy and Bernoulli equations, which account for changes in the kinetic energy as a result of a cross–sectional flow area change. They are calculated from the following equations.

$$K_{B_{inlet}} = 1 - (d/D)^4 \tag{26a}$$

$$K_{B_{outlet}} = (d/D)^4 - 1 \tag{26b}$$

Thus, if reducers of identical size are used at the inlet and outlet, these terms cancel out.

The term K_I in equation 25 includes the loss coefficients and Bernoulli coefficient on the inlet side only.

In the absence of test data or knowledge of loss coefficients, loss coefficients may be estimated from information contained in other resources such as reference 3.

The factors F_p and F_{lp} would appear in flow equations (15) and (21) respectively as follows:

For incompressible flow:

$$Q = F_p C_v \sqrt{\frac{P_1 - P_2}{G}} \tag{27}$$

For choked flow:

$$Q_{\max} = F_I C_v \sqrt{\frac{P_1 - F_F P_v}{G}} \tag{28}$$

Summary

It has been shown that a fundamental relationship exists between key variables (P_1, P_2, P_v, G, C_v, Q) for flow through a device such as a control valve. Knowledge of any four of these allows the fifth to be calculated or predicted. Furthermore, adjustments to this basic relationship are necessary to account for special considerations such as installed piping configuration, cavitation, flashing, choked flow, and viscous flow behavior. Adherence to these guidelines will insure correct sizing and optimum performance.

References

1. Streeter, Victor L., and E. Benjamin Wylie, *Fluid Mechanics*, 7th Ed., McGraw–Hill Book Company, New York, 1979.

2. Olson, Reuben M., *Essentials of Engineering Fluid Mechanics*, 3rd Ed., Intext Educational Publishers, New York, 1973.

3. *Flow of Fluids Through Valves, Fittings, and Pipe*, Crane Company, New York, 1978.

4. Instrument Society of America, ‘‘Control Valve Capacity Test Procedure’’, ANSI/ISA–S75.02, 1981, Research Triangle Park, North Carolina.

5. Instrument Society of America, ‘‘Control Valve Sizing Equations’’, ANSI/ISA–S75.01, 1977, Pittsburgh, Pennsylvania.

6. Schafbuch, Paul M., *Fundamentals of Flow Characterization* (*Technical Monograph 29*), Fisher Controls International, Inc., Marshalltown, Iowa, 1985.

7. *Control Valve Handbook*, Fisher Controls Company (Fisher Controls International, Inc.), Marshalltown, Iowa, 1977.

8. Stiles, G.F., ‘‘Development of a Valve Sizing Relationship for Flashing and Cavitation Flow’’, Proceedings of the First Annual Final Control Elements Symposium, Wilmington, Del., May 14–16, 1970.

9. Stiles, G.F. ‘‘Liquid Viscosity Effects on Control Valve Sizing’’, 19th Annual Symposium on Instrumentation for the Process Industries, Texas A & M, 1964.

Fisher Controls International, Inc.
205 South Center Street
Marshalltown, Iowa 50158 USA
Phone: (641) 754-3011
Fax: (641) 754-2830
Email: fc-valve@frmail.frco.com
Website: www.fisher.com

D350408X012 / Printed in U.S.A. / 1985

FISHER-ROSEMOUNT™